Name Key

Polynomial Review Sheet #1

1. Write the equation of a cubic function in standard form that has roots at {-1, 2, -3} and passes through the point (3, 48).

$y = a(x+1)(x-2)(x+3)$

$48 = a(3+1)(3-2)(3+3)$

$48 = 24a$

$a = a = 2$

$y = 2(x+1)(x-2)(x+3)$

$x^2 - 2x + x - 2$

$2(x+3)(x^2 - x - 2)$

$2(x^3 - x^2 - 2x + 3x^2 - 3x - 6)$

$2(x^3 + 2x^2 - 5x - 6)$

$y = 2x^3 + 4x^2 - 10x - 12$

2. Given the following equation: $y = x^5(x+2)^3(x-3)^2$

   a. State the degree of the polynomial

   10

   b. State the zeros and the multiplicity of each zero

   $x = 0$ mult. 5 $x = -2$ mult. 3 $x = 3$ mult. 2

3. Use synthetic division to find the quotient and remainder of: $\frac{4x^3+15x^2-9x-18}{x+2}$

-2	4	15	-9	-18
		-8	-14	46
	4	7	-23	28

$4x^2 + 7x - 23 + \frac{28}{x+2}$

4. Use long division to divide $2x^4 - 9x^2 - 10x - 3$ by $x^2 + 2x + 1$

$x^2 + 2x + 1 \;\overline{)\; 2x^4 + 0x^3 - 9x^2 - 10x - 3}$

$2x^2 - 4x - 3$

$-(2x^4 - 4x^3 - 2x^2)$

$-4x^3 - 11x^2 - 10x$

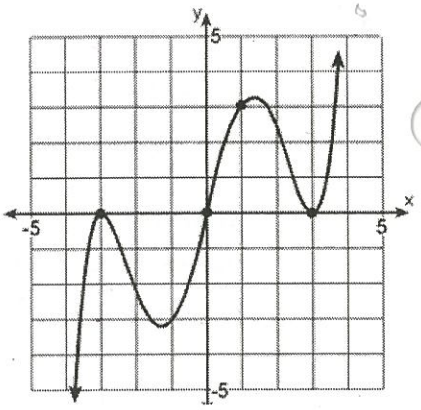
$-(-4x^3 + 8x^2 + 4x)$

$-3x^2 - 6x - 3$

$-(-3x^2 + 6x + 3)$

$2x^2 - 4x - 3$

5. Below is a graph of a 5th degree polynomial.



a. State the zeros and the multiplicity of each

$x = -3$ mult. 2 $x = 0$ mult. 1 $x = 3$ mult. 2

b. Determine an equation of the polynomial in factored form.

(1, 3)

$y = a(x+3)^2(x+0)(x-3)^2$

$3 = a(1+3)^2(1+0)(1-3)^2$

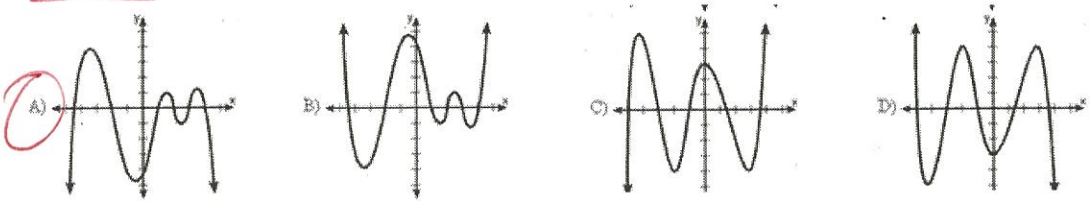
$3 = 64a$

$a = 3/64$

$y = \frac{3}{64}(x+3)^2(x+0)(x-3)^2$

$y = \frac{3}{64}x(x+3)^2(x-3)^2$

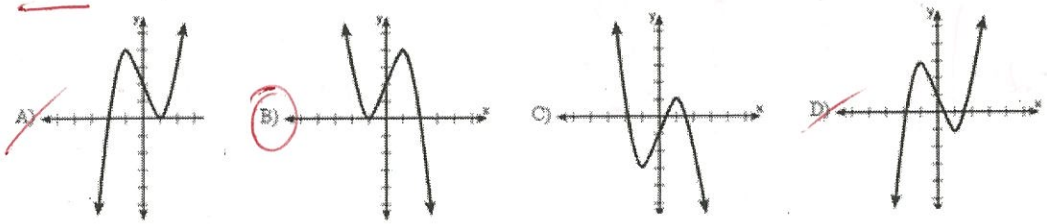
6. Which of the following best illustrates a 6th degree polynomial whose leading coefficient is negative?



7. Which of the following graphs best illustrates the graph of $y = a(x-b)^2(x-c)$ where $a < 0$ and $b \neq c$.

bounce

odd degree



8. Find the remainder when $P(x) = 5x^4 - 20x^2 - 84$ is divided by (x + 5).

$5(-5)^4 - 20(-5)^2 - 84$

$= 2541$

9. Given $P(x) = x^3 + kx^2 - 11x - 6$, find k if (x-2) is a factor of P(x).

→ remainder = 0

$(2)^3 + k(2)^2 - 11(2) - 6 = 0$

$4k - 20 = 0 \rightarrow 4k = 20$

$k = 5$

10. Determine if (x – 2) is a factor of $x^5 - 2x^3 - x^2 + 2$. Explain your reasoning.

$(2)^5 - 2(2)^3 - 2^2 + 2 = 14$

No because there is a remainder

11. Algebraically determine the zeros of $x^7 - 8x^5 + 16x^3 = 0$ and state their multiplicity.

$x^3(x^4 - 8x^2 + 16) = 0$

$x^3(x^2 - 4)(x^2 - 4) = 0$

$x = 0 \quad x = \pm 2 \quad x = \pm 2$

$x = 0$ mult. 3

$x = 2$ mult 2

$x = -2$ mult 2

12. If $P(x) = ax^7 + x^6 - bx^2 - 4x + c$, where a, b and c are positive values, describe the end behavior of P(x).

As $x \to \infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to -\infty$.

degree = 7

a positive

13. If $x + 2$ is a factor of $f(x) = 2x^3 + 9x^2 + 7x - 6$, algebraically find all of the zeros of $f(x)$.

	2	9	7	-6
-2		-4	-10	6
	2	5	-3	0

$2x^2 + 5x - 3$

-6, +6x, -1x, 5

$2x^2 + 6x - 1x - 3 = 0$

$2x(x+3) - 1(x+3) = 0$

$(2x-1)(x+3) = 0$

$x = 1/2 \quad x = -3$

$\{-3, -2, 1/2\}$

14. Prove the polynomial identity: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$

$(x+y+z)(x+y+z)$

$x^2 + xy + xz + xy + y^2 + yz + xz + yz + z^2$

$x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$

use calculator

15. Which of the following identities is incorrect?

(1) $(a-b)^2 = a^2 - 2ab + b^2$ ✓

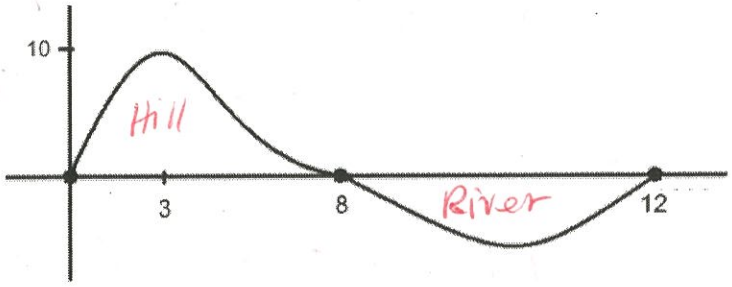
(2) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ ✓

(3) $r^9 - s^2 = (r^3 - s)(r^3 + s)$ ✓

(4) $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$

16. Geographers sit at a café discussing their field work site, which is a hill and a neighboring riverbed. The hill is approximately 10 feet high, 8 feet wide, with a peak about 3 feet east of the western base of the hill. The river is about 4 feet wide. They know the river is shallow, no more than about 2 feet deep.

They make the following crude sketch on a napkin, placing the profile of the hill and riverbed on a coordinate system with the horizontal axis representing ground level.



The geographers do not have any computing tools with them at the café, so they decide to use pen and paper to compute a cubic polynomial that approximates this profile of the hill and riverbed.

a. Find the x-intercepts $x = 0, 8, 12$

b. State the domain that would represent the hill. State the domain of the riverbed.

Hill: $(0, 8)$ River: $(8, 12)$

c. Find the relative maximum.

$(3, 10)$

d. Write a cubic polynomial function H that could represent the curve shown (here, x represents the distance, in feet, along the horizontal axis from the western base of the hill, and $H(x)$ is the height, in feet, of the land at that distance from the western base). Be sure that your formula satisfies the relative maximum of curve. (Round any decimals to the nearest *thousandth*)

$10 = a(3+0)(3-8)(3-12)$

$10 = 135a$

$a = .074$

$H(x) = .074x(x-8)(x-12)$